

Insight 2015

Smith *institute*
for industrial mathematics and system engineering

**Accelerating innovation
and building better
businesses using the
mathematical sciences**

Harnessing mathematics

It is now more than 20 years since the Smith Institute was first established, and throughout that time its mission has remained constant: to improve the world through harnessing mathematics more effectively.



In the past year, we have seen rapid development in the size and scope of our projects. The work we undertake for clients ranges from the development and implementation of research strategies right through to the final deployment of business-critical systems. The common thread that binds all these activities together is their central reliance on using the right mathematics in the right way at the right time.

In support of the successful exploitation of new business opportunities, we have created a Scientific Board of leading academics, for the exchange of commercial and scientific ideas. We have also made the first two appointments to the new position of Vice-President. Professor Frank Kelly and Lord Julian Hunt are advocates of the utilisation of mathematics and will provide strategic advice to the Smith Institute. These relationships promote the early development of new opportunities and help us make fresh connections with existing work. In short, we are working to create a science and innovation environment in which mathematics can flourish, to the benefit of business.

This report gives a flavour of the range of work we do and the people who do it. The Smith Institute now has 20 staff and its continued success is built around their energy and ambition. We take a long-term view, while maintaining the ability to respond to short-term needs, underpinned by an annual growth of 10-15%.

We hope you enjoy reading about the Smith Institute and look forward to hearing from you.

A handwritten signature in black ink, appearing to read 'R Leese'. The signature is fluid and cursive, written on a white background.

Dr Robert Leese
Chief Executive, Smith Institute

Outlook

Mathematics is growing in prominence as an essential component for building better businesses and better public services and infrastructure. Organisations that are able to bring a mathematical way of thinking to high-priority challenges will find an increasing demand for their services. In part, this trend is being driven by the data revolution, but mostly by the growing awareness in business, government and academia that mathematical skills are highly transferable and rapidly deliver value when put to effective use.

These shifting views are far from confined to the UK. The Smith Institute is gaining new clients and collaborators in Europe, North America and Asia, especially around applications where we have established track records. Tapping into these opportunities, both in the UK and overseas, will require growing the industrial mathematics

community at large. Greater connectivity across the science and innovation environment will enable new technologies and services to progress from initial design through proof-of-concept to successful deployment. Being successful will require people who are able to work across all stages of the process.

The Smith Institute is committed to building activity and capacity in this space. We will continue to work with universities to help them build wider strategic relationships with business; we will continue to work with government agencies to ensure that industry and society benefits from public investment in mathematics; and most importantly we will work with the huge diversity of companies with whom we can unlock the full economic value of the work we do.

The Smith Institute story so far

From lunar landings to using applied mathematics to boost business.



It all began fifty years ago, in 1965, when our founder, Dr Bruce Smith CBE, took a job at Bellcomm Inc in Washington DC in the manned space-flight program. Bruce had moved to the US in 1964 to work in physics research at the University of Chicago following his DPhil in theoretical physics at Oxford. Bellcomm carried out system engineering for the Apollo programme and Bruce was employed to work in a team selecting landing sites for the man on the Moon.

System engineering as practised in Bellcomm was the design of major engineering systems from the top down, rather than from the bottom up as was then common practice in engineering, particularly in the UK. It ensures that all aspects of a project or system are considered and integrated as a whole. It overlaps technical and human centred disciplines such as control, industrial and software engineering with organisational studies and project management.

Bruce was very taken by his experience at Bellcomm and realised that there might be a business opportunity to create a system engineering company back in the UK.

He became inspired to apply mathematical techniques to industry and government problems and after returning to the UK and working for a spell at Decca Radar in military electronic systems, Bruce started a consulting system engineering company in 1971 with the name Smith Associates, which later became Smith System Engineering Ltd.



And so, Bruce set to work in a room above his lawyer's office in Weybridge, establishing the concept of a consulting system engineering company and finding clients. He worked long hours alone, building the turnover of the company slowly until he was able to hire his first employee in 1974. By 1979 he employed a team of ten. Together their work consisted of designing elements of major engineering systems for large clients in areas such as radar, communications and advanced computing. Their clients included Decca Radar, the Ministry of Defence and BP.

The company grew steadily and organically over the years and the client base expanded rapidly. In 1993, having moved into offices over shops in Cobham and then taking a building on the Surrey Research Park in Guildford, Bruce formed the Smith Institute for Industrial Mathematics and System Engineering as a division of Smith System Engineering Ltd. The new division was set up to help formalise links with universities.

A few years later, in 1997, Bruce sold Smith System Engineering Ltd (now employing 150 staff) without the Smith Institute for Industrial Mathematics and System Engineering, by means of a financed

management buy-out. He left the organisation and set up the Smith Institute as a separate independent company.

The mix of clients and industrial sectors and the Institute's close working relationship with academia increased and broadened a great deal from 1997 onwards and the Smith Institute is now well established as one of the leaders of the applied mathematics community in the UK.

Bruce is now President of the Smith Institute and remains involved and interested in its work.

The team includes 16 talented consultant mathematicians with a variety of strong academic backgrounds and a wide range of industrial experience. We are proud of our work with industry and government and our strong connections with the academic mathematics community. Mathematics underpins everything we do because we know it can really boost business and accelerate growth.

We look forward to the next fifty years!

The team



**Robert
Leese**



**Andrei
Bejan**



**Anna
Railton**



**Caroline
Edwards**



**Cristina
Sargent**



**David
Miller**



**Jakob
Blaavand**



**Gillian
Hoyle**



**Heather
Tewkesbury**



**Tim
Boxer**



**Paul
Munday**



**Vera
Hazelwood**



**Michelle
Ledbetter**



**Claudia
Centazzo**



**Judy
Reynolds**



**Melvin
Brown**



**Torran
Elson**



**Zoe
Kelson**



**David
Allwright**

Performing mathematics

We think about problems using mathematical concepts and language and combine these with our knowledge of system design and system engineering. Most of our projects use the following inter-related components:

Mathematical models

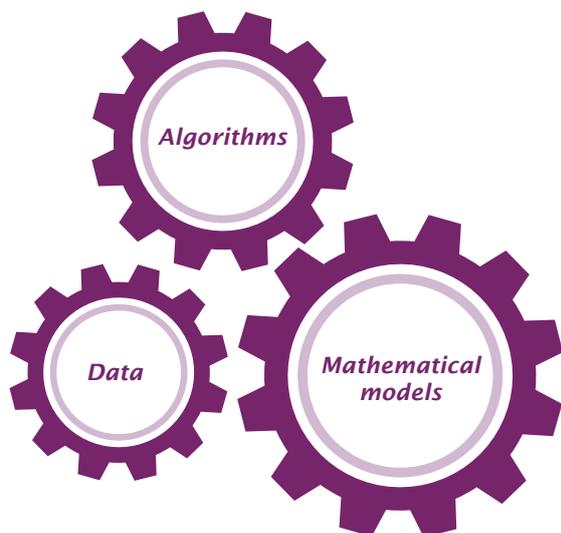
Models provide a rigorous way of analysing a problem and finding solutions. They allow us to test different scenarios to see the evolution of systems and provide a quantitative analysis of outcomes. The results can underpin decision making and policy formulation.

Interrogation of large sets of data and integration into models

Mathematics provides the conduit for extracting knowledge from varied and complex information.

Algorithms

Algorithms are the sets of step-by-step operations which perform calculations, data processing and reasoning in order to solve the problem.



Choosing the right combination of models, data and algorithms provides the key that unlocks the value in the way we work.

Solution implementation



vodafone

Spectrum auction bids reconstruction



Interferogram analysis to aid interpretation of experimental data

TESCO

Online general merchandise prices



Weight estimation methods for aircraft



Decision support for validation and verification of future aircraft



Exploring new concepts and technologies



Detecting illegal fishing

nationalgrid

Forecasting peak gas demand

Forecasting peak gas demand for the UK future energy scenarios

The Smith Institute independently reviewed the technical basis of the 1 in 20 peak demand forecasting methodology for National Grid.

The problem

Every year at National Grid, the National Transmission System (NTS) team provides the Network Strategy team with the year-ahead annual gas demand forecasts. These forecasts are used by the Network Strategy team to book gas supply capacity to meet the UK's year-ahead gas demand for the whole year. Understanding the accuracy and robustness of these demand forecasts is critical to the Network Strategy team's business.

The Smith Institute was asked by National Grid to conduct an independent review of the technical basis of the 1 in 20 peak demand forecasting, and so underpin the confidence of the Network Strategy team in the use of such forecasts for booking year-ahead gas capacity.

The Smith Institute found that the methodology used to generate the 1 in 20 peak demand forecast is fit for purpose, and suggested enhancements to improve its performance.

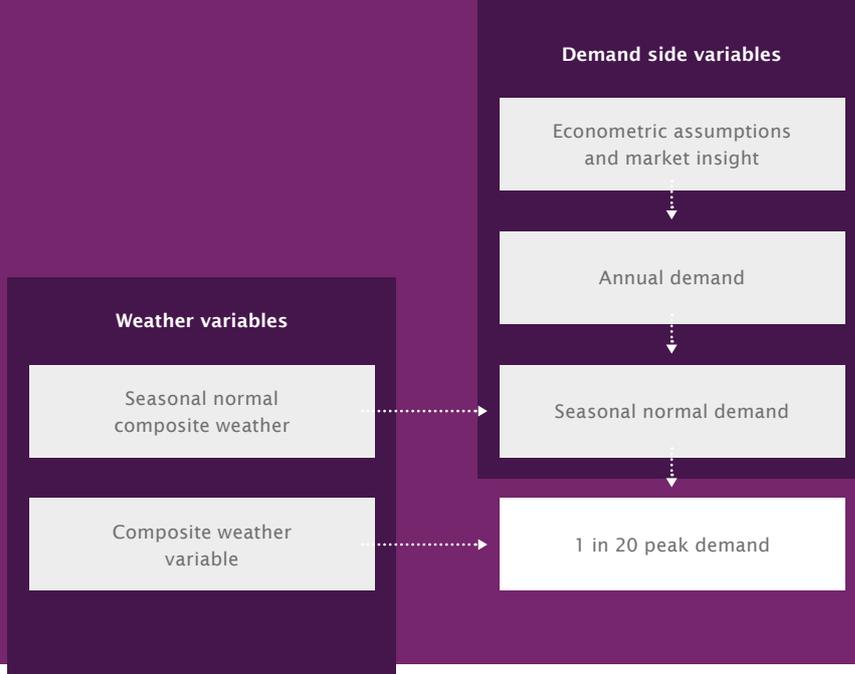
The solution

To assess the methodology used for generating the 1 in 20 peak demand forecast and to identify potential enhancements to the existing methodology, the Smith Institute independently developed mathematical and statistical models and applied them to data provided by the NTS and Network Strategy teams. Outcomes from the modelling and analysis were regularly reported to and discussed with both teams to gain shared insights.

Our analysis comprised the following key stages:

- reviewing and scrutinizing each step in the documented methodology used to generate peak gas demand forecasts;
- developing models for demand forecasting that make effective use of raw historical data;
- testing relationships between variables in the existing methodology;
- validating data-related and operational assumptions in the existing methodology; and
- developing an understanding of why recent demand forecasts from NTS have been perceived to be high by the Network Strategy team.

To conclude the project, we held a workshop at National Grid to present our findings, to discuss the feasibility of their application and to provide the opportunity for feedback from National Grid. During the workshop, we highlighted aspects of the methodology that need improving, and categorised our recommendations into those for immediate improvement to the 1 in 20 peak estimates and those for implementation in the medium to long-term as part of planned improvements to National Grid's demand forecasting methodology. Interesting discussion arose over the validity of the assumption that National Grid should always base their forecasts on the 1 in 20 peak, particularly as we had demonstrated the sensitivity of peak estimates to composite weather variable history: a longer history results in higher peaks.



The benefit

Both the NTS and Network Strategy teams found the analysis valuable in evaluating and further underpinning the robustness of the process used by National Grid.

The causal links between variables that shape peak demand are shown in the picture above, which illustrates the stages in the demand forecast methodology used by National Grid.

The Smith Institute's key findings can be summarised as follows:

- different load bands respond in different ways to economic growth scenarios. For example, in the Going Green scenario domestic load is particularly responsive, whereas in the Slow Progression economic growth scenario the demand in the SME segment is significantly affected;
- peak demand forecasts are significantly influenced by economic assumptions within future energy scenarios for gas; and
- Composite Weather Variable (CWV) history has a significant impact on peak demand estimates: specifically, longer history results in higher peaks.

The Smith Institute is confident that the methodology used to generate the 1 in 20 peak demand forecast is fit for purpose, although improvements can be made to enhance its performance. In particular, we were able to suggest:

- improvements to the modelling of the peak demand distributions;
- improvements to the modelling of the random error term in the demand versus CWV regression model; and
- a novel formulation of the demand versus CWV relationship based on support vector regression.

Since the completion of our analysis, the NTS team has implemented the suggested improvements to the random error term for the CWV regression model. NTS is also planning to include the Smith Institute's recommendations into National Grid's changes for next year's demand forecast methodology, in preparation for creating next year's year-ahead forecasts. Our comparison between the current (Gumbel) distribution and our proposed (Weibull) distribution, for estimating the likelihood of different peaks, showed that the proposed distribution yielded higher confidence in the forecasts of 1 in 20 peaks over longer horizons. NTS intends to capitalise on these findings.

"The review by the Smith Institute was both rigorous and professional, providing both assurance and furthering learning for our business. The study has provided a timely input and supportive of our needs and the Smith Institute were a pleasure to work with."

Dr Stephen Marland

Gas Demand Manager at National Grid

How much will the next generation Airbus aircraft weigh?

The Smith Institute helped leading aircraft manufacturer Airbus to review their weight estimation methods for aircraft weight outlook.

The problem

The design phase for a new aircraft is a long process that can span several years. Materials and technologies evolve during the lifespan of the design, so Airbus needs to monitor the aircraft weight (status mass) at each milestone review and to estimate what the aircraft will weigh (weight outlook) throughout the process. The introduction of new materials and new technologies, and changes in standards and regulations, are all factors that may affect the aircraft weight; each needs to be carefully considered and modelled in estimating the weight outlook.

Airbus wished to review the weight estimation methods for an aircraft during the design phase and to ensure that the derivation of weight estimates and their associated uncertainties are underpinned by sound mathematical and statistical methods.

The solution

The Smith Institute organised and ran an initial one-day workshop at Airbus premises in Filton with members of the Airbus Mass Estimation and Control Team, to explore their use of statistics and mathematics in modelling mass properties. Through the use of examples, Airbus staff described their objectives and their application of statistical methods to weight estimation, so enabling the Smith Institute to review them and to explore component weight trades and sensitivities.



The Smith Institute proposed new methods and established the feasibility of their application by direct discussion with Airbus staff. In the course of the workshop, we considered the following:

- the interaction between components;
- sensitivity analysis – prioritisation;
- uncertainty estimation methods;
- aggregation – Monte Carlo simulations, and
- Bayesian methods for obtaining weight and parameter estimates and their uncertainties.

After the workshop, we articulated enhancements, and proposed new methods for weight estimation and its communication to the design team. These findings were discussed with Airbus in a second one-day workshop and were presented in a report of our review and recommendations for enhancing Airbus weight estimation methods.

The benefit

The Smith Institute provided an independent analysis of the weight estimation methods and helped Airbus enhance the rationale behind the methods currently used.

We provided different ways of looking at the weight estimation process, introducing the Airbus team to alternative methods of data fitting, of goodness of fit evaluation, and of ways of presenting uncertainties in data.

By working on practical examples based on Airbus models, the Airbus team and the Smith Institute were able to discuss the methods in great detail, highlighting opportunities to refine and improve the Airbus weight estimation process.

“I very much appreciated the time taken, the patience shown and the ability of the team to explain things in simple enough terms using examples so we were not bamboozled by the maths.”

Judi Cheeseman
Airbus Operations Ltd

Playing by the rules: getting auctions right

Since 2007, the Smith Institute has worked with clients around the globe on implementing auctions for radio spectrum licences.

Bearing in mind that auction outcomes can shape a country's mobile telecommunications industry for a decade or more, the tasks of getting the auction designs right and running auctions without a hitch leave no room for mistakes.

There is always extensive discussion and consultation over proposed auction rules, and of course one would expect major industry players to speak up for rules they consider favourable to their own interests. But what are the real underlying issues?

Imagine there is to be an auction of fine furniture at your local saleroom, and there is a nice table and also a set of chairs that have caught your eye. Just the thing for that important anniversary coming up. You have £1,000 to spend (from matching 5 balls in last Saturday's lottery draw). The only problem is that the table and chairs are being sold as separate lots. You go along to the sale, and the table comes under the hammer first. There are a few other bidders interested and the bids start to rise ... £500, £550, £600, ... The chairs have a reserve price of £250 and you think they might sell for about £350, but that's only a guess - and the excitement seems high in the room today. Do you keep bidding on the table or not?

You might stop bidding on the table, but how would you feel if it finally sold at £750 and then you find out a few minutes later that no-one else is interested in the chairs, when they come up for sale? Or you might carry on and win the table for £700, but then find that bidding on the chairs is

just as fierce. In that case, you have to go home with just a table and carry on playing the lottery, hoping to come back to another sale soon.

What you would really like to do is to communicate to the auctioneer that you wish to bid up to £1,000 for both table and chairs, but the auction is not designed that way. This is an example of the exposure problem. Combinatorial auctions remove the exposure risk. This means that single bids can be placed on packages of items, in this case a package consisting of the table and chairs. Another bidder might be interested in the same set of chairs and the fine bookcase (which caught your eye too, but for the fact you already have one). In general, different bidders will be interested in different, but overlapping packages, and an individual bidder will generally bid on multiple packages at different amounts.

The construction of combinatorial auctions is now well established and there are numerous examples of their successful implementation. In addition to avoiding the exposure problem, they have high economic efficiency, meaning that in applications like radio spectrum their outcomes tend to be ultimately good for consumers. They also create strong incentives for bidders to be truthful and consistent in the bids that they place. For bidders, the main task is to decide which packages are of interest and at what valuations, and these decisions depend on being clear about their individual business plans.

One of the challenges is in managing the potential combinatorial 'explosion'. The UK auction for 4G spectrum in February 2013 sold licences for



28 individual 'lots' of spectrum, which could be assembled into about 15,000 possible packages. On a different scale was the Canadian auction in the 2500MHz band, in the spring of 2015. This auction had 318 lots and in theory more than 10^{39} possible packages.

Canada's auction had a more detailed lot structure to reflect the regional structure of its telecommunications market. Spectrum licences related to a particular 'lot' of frequencies but also to a particular geographical region - 61 regions in Canada's case. Regional service providers and local service providers were able to participate alongside national operators, in the same auction process. Each participant was able to concentrate on those packages that were compatible with its own business model, which will be some tiny fraction of the full 10^{39} . This auction also broke new ground in the way bidders express their preferences, to handle such a large number of possibilities.

Spectrum auctions are conducted using secure electronic systems and may easily last several weeks, with multiple rounds of online bidding. Depending on the number of lots and how they can be combined into packages, we use different mathematical approaches to verify the results. The main tasks are identifying winners and prices from what might easily be several thousand bids in total.

If the number of possible packages is relatively low, say up to a few million, then the mathematical approach of 'dynamic programming' is a good choice, and works by building up the overall solution one bidder at a time. For larger auctions, we use 'integer programming', which is a more

general-purpose approach to solving combinatorial problems (for example it is widely used to solve scheduling problems, including the famous travelling salesman problem). We have also developed a set of automated and manual checks on the results, which are founded on the underlying mathematical theory of optimisation problems, and allow us to be completely certain that the declared results are correct.

Listening to the press and other industry commentators, it's easy to pick up the idea that auction implementation has somehow got out of hand compared with the local saleroom. The Guardian newspaper reported during the UK 4G auction in 2013 that 'The outcome ... is being determined by secret mathematical algorithms worked out using massive computer processing power'. This wasn't true for the UK auction and it isn't true for subsequent auctions. Chi Onwurah, currently the MP for Newcastle Central and formerly senior technologist at Ofcom was mischievously quoted as saying '[The auction rules] are something that only 10 people in the world understand and half of them are mad.' As far as we know, that's not true either!

What is clear, however, is that the latest developments in auction design and implementation are helping to ensure that telecommunication markets can keep pace with the seemingly insatiable demand for mobile services.

When, where and how? Planning mass transport

Transport systems are subject to uncertainties that must be managed both in planning and in operations.

Uncertainties arise from fluctuations in demand, unexpected disruptions, maintenance, available investment capital, regulatory uncertainty etc. In such an environment, it is a major challenge to make decisions that optimise the balance between costs and user/social benefits.

The Smith Institute has embarked on the development and demonstration of a software framework that will make new approaches available to transport practitioners. With assistance from the Department for Transport, in the form of a Technology Research Innovation Grant, we will focus initially on a technique known as Approximate Dynamic Programming (ADP). At present, the use of ADP is confined mostly to the research community, although with some limited industrial deployment.

ADP is particularly suited to situations where decisions have to be made sequentially and the outcomes are not visible on short timescales, i.e. decisions can influence benefits across multiple decision points. It is also a very flexible technique, potentially elucidating optimal decision strategies for problems including resource allocation, capital investment and tariff structures.



We intend to make the results of this work freely available and to support early adopters in exploiting the possible cost savings and service improvements. The benefits should be widely accessible, and we anticipate these same techniques also being readily adaptable to applications in the energy sector.

A week in the life of a mathematician

People often ask us, “What does a mathematician do at work?”



I'm Torran, a consultant mathematician at the Smith Institute.

I mostly work from home, which offers the best of both worlds: I'm afforded the precious quiet

space to really get to grips with the technical issues of a project, whilst knowing I have the support of my fellow colleagues who are only a phone call away. This week I've got a team meeting at our Harwell office on Monday and a conference on Friday in Cambridge; Tuesday to Thursday I'll be at home.

The team meeting starts with a structured session where each member of staff gives a brief update on their work and asks for input from others, if needed. Robert, the CEO, and Heather, Director of Business Development, inform us of any strategic issues that we need to know about. This is one of the best things about working in a small company – you're involved and aware of decisions up to the strategic level. Next is a less structured session where we have discussions based around project work. I'm working on a data project with Paul. Together we go through the progress we've made and make a plan for what we should achieve this week.

Technical work follows a pattern that is probably familiar to many mathematicians: considering the different ways of tackling a problem, drinking coffee, assessing what might be the best solution approach for the job that needs doing, calling a colleague to draw on their advice, deciding which route to follow and then pulling all of the details together to get something that works. I spend much of Tuesday implementing the regression analysis that I discussed with Paul yesterday. Things go smoothly and I get a working script written in R.

With the regression script for the data project working, Paul and I spend Wednesday writing up the results. We use Skype to discuss our progress and during the call we start collaboratively editing a document. We work like this often: it makes the process of writing a report quicker and more enjoyable.

With a conference tomorrow, Thursday is about finishing things up for the week. I edit the report on our regression analysis and Paul and I get the report finished with time to spare.

Friday's conference is on “Big Data in Medicine.” Vera and I are attending with the aim of finding mathematical challenges which the Smith Institute can help address.

Eight talks cover a range of topics from how to link patient records so a full medical history can emerge, to mining linked databases to find new drugs. In the final few talks a number of PhD students discuss their posters on mathematical biology. These are fascinating as they apply familiar techniques to entirely novel applications.

On the journey home I write up my notes from the conference – what I have learnt, what the main themes were, and what I think the mathematical challenges are. This will inform a presentation Vera and I will give at the next team meeting.

Now I'm ready for the weekend after a rather busy week, I had one face-to-face meeting, three teleconferences, one webinar, and one conference; produced one working piece of code; and helped write a report!

Smith Institute Scientific Board



John Ockendon

Professor of Applied Mathematics,
University of Oxford

John is the Smith Institute's Chief Mathematician and Chairman of its Scientific Board. He helped to found the Oxford Centre for Industrial and Applied Mathematics (OCIAM) and is stimulating research in the area of Free Boundary Problems. John built his career and international activity around mathematics-in-industry, beginning with the first Maths-in-Industry Study Group in 1968.



David Abrahams

Professor of Applied Mathematics,
University of Manchester

David has made substantial contributions to advances in underwater acoustics, aeroacoustics, optics, elasticity and more recently mathematical finance. He is currently adapting matched asymptotic expansions to 'messy' industrial wave problems and is also involved in the development of models of pre-stressed materials which have wide application to industrial and biological components.



Paul Munday

Consulting mathematician,
Smith Institute

Paul is interested in the mathematical and statistical techniques for correctly fusing expert judgement and prior knowledge with limited data for the purpose of decision making.



Richard Craster

Professor of Applied Mathematics,
Imperial College London

Richard has made extensive contributions to the field of Fluid Mechanics, including identifying the physical origin of instabilities in surfactant-driven flows and superspreading. He is currently researching wave propagation in metamaterials to protect buildings from ground vibration.

Helping business and industry to make decisions effectively is one of our core services. This means imparting a way of thinking: to value both the data available and the expert judgement; and to accept the uncertainties of a situation while still making a quantitative assessment. The tools needed to do this can be found in many places. Intelligent modelling of a problem is always critical and the Smith Institute keeps in touch with the latest advances in this area.



Mark Girolami

Professor of Statistics,
University of Warwick

Mark's research focuses on Computational Statistics. He is working with various companies, including Amazon and Microsoft, on problems mainly of a statistical nature. His work has led to patented technologies which have been deployed in commercially successful products and services. Mark is also working on Pattern Recognition and Predictive Methods.



Alan Champneys

Professor of Applied Non-linear Mathematics, University of Bristol

Alan's work spans the areas of applied nonlinear dynamics, global bifurcations, pattern formation, non-smooth dynamics, engineering mechanics, optics and physiology. He has a growing number of industrial collaborations, focusing on problems connected to rotating machines, valve instability, friction and impact modelling and the dynamics of supply networks.



Richard Weber

Professor of Mathematics for Operational Research, University of Cambridge

Richard has made significant contributions to theory in communications and operations management, control of queues, stochastic networks, optimal search and dynamic resource allocation. He has worked on industrial projects which involved optimising systems for insurance, the food industry, and waterways.

David Allwright

Consulting mathematician,
Smith Institute

David has modelled numerous and varied products, processes and systems for industry and government.

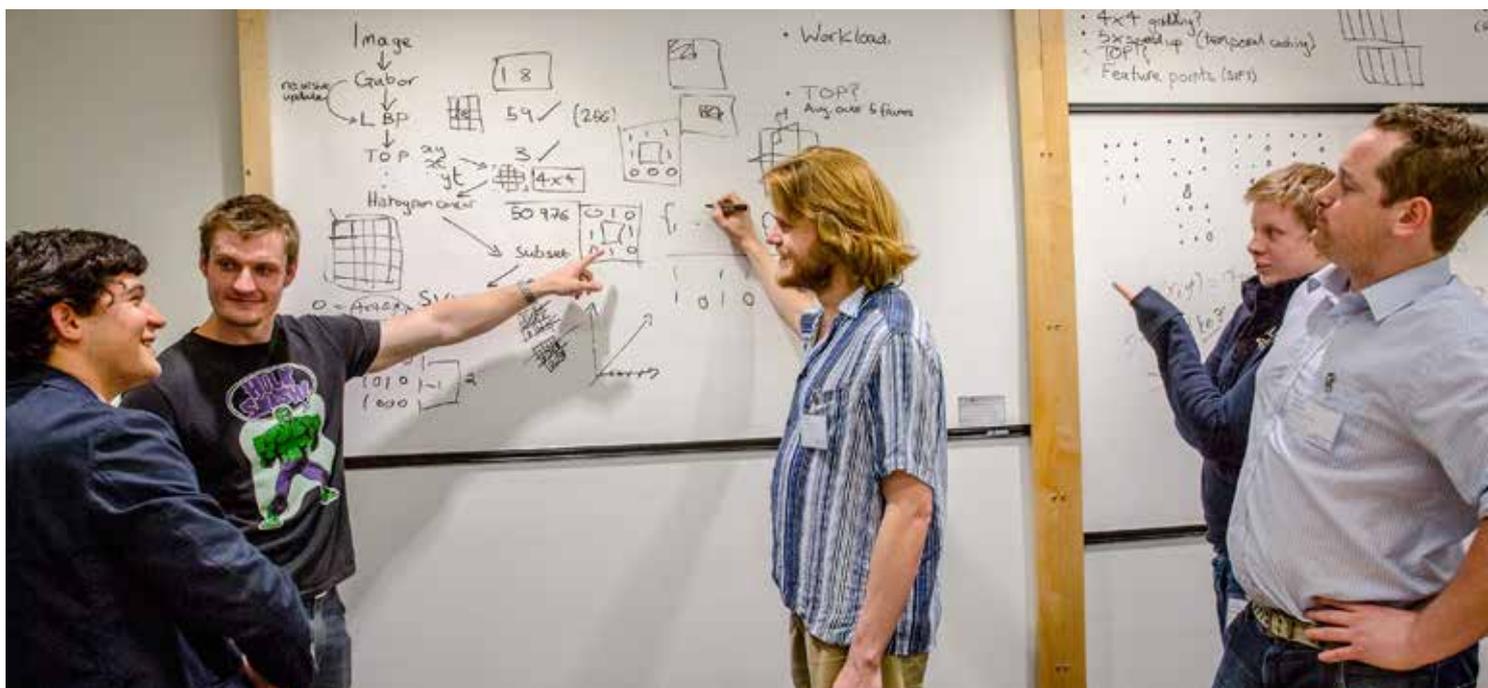
The potential for mathematicians to respond usefully to the needs of industry, commerce and government is boundless.

The ability of mathematicians to provide models at a level of detail appropriate to the problem is very powerful. For problems in the traditional areas of applied mathematics, this may involve analysis and computation using classical models, but often assembled together in new ways to answer new questions. For problems in novel areas, the development of appropriate new models that capture the level of detail appropriate to the problem, and analysis and computation based on those models can provide insights and results available in no other way.

Stimulating mathematics

The Smith Institute values its connections with academia and promotes projects that benefit all parties by solving industrial problems, stimulating new mathematics and creating productive relationships.

We build our network of industrial and academic partners by continuously creating new collaborative projects and opportunities via events, workshops and study groups both in the UK and abroad. We facilitate and manage cross-sector collaborations for a number of clients, including Innovate UK, Knowledge Transfer Network (KTN Ltd) and the Natural Environment Research Council's Probability, Uncertainty & Risk in the Environment (PURE) Network.



Industrial Cooperative Awards in Science & Engineering (iCASE)

This spring we allocated four Engineering and Physical Sciences Research Council (EPSRC) iCASE studentships to companies and universities to support postgraduate researchers working on new collaborations in mathematics:

Stochastic modelling of intra-cellular bacterial infections

The Defence Science and
Technology Laboratory with
University of Leeds

Topology optimization under uncertainty for gas turbine internal flows

General Electric Oil & Gas
with Imperial College London

Efficient inverse uncertainty quantification for partial differential equation models with uncertain data

National Physics Laboratory
with University of Manchester

Linking physiologically based pharmacokinetics, mathematical modelling and systems biology

GlaxoSmithKline with
University of Strathclyde

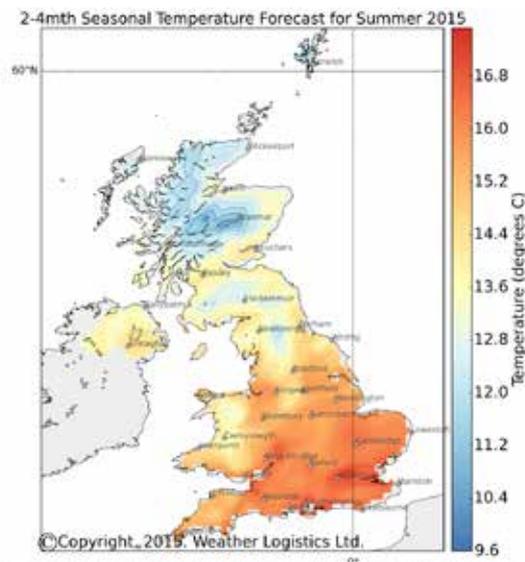
European Study Group with Industry

The Smith Institute organises the annual UK ESGI (European Study Group with Industry), bringing mathematicians and industrialists together over an intensive week-long event to work side by side and solve the real and important issues that companies are facing today.

Weather Logistics, a recent Nottingham start-up in the field of long-range forecasting, attended the 107th European Study Group with Industry (ESGI107), hosted by the University of Manchester in March 2015.

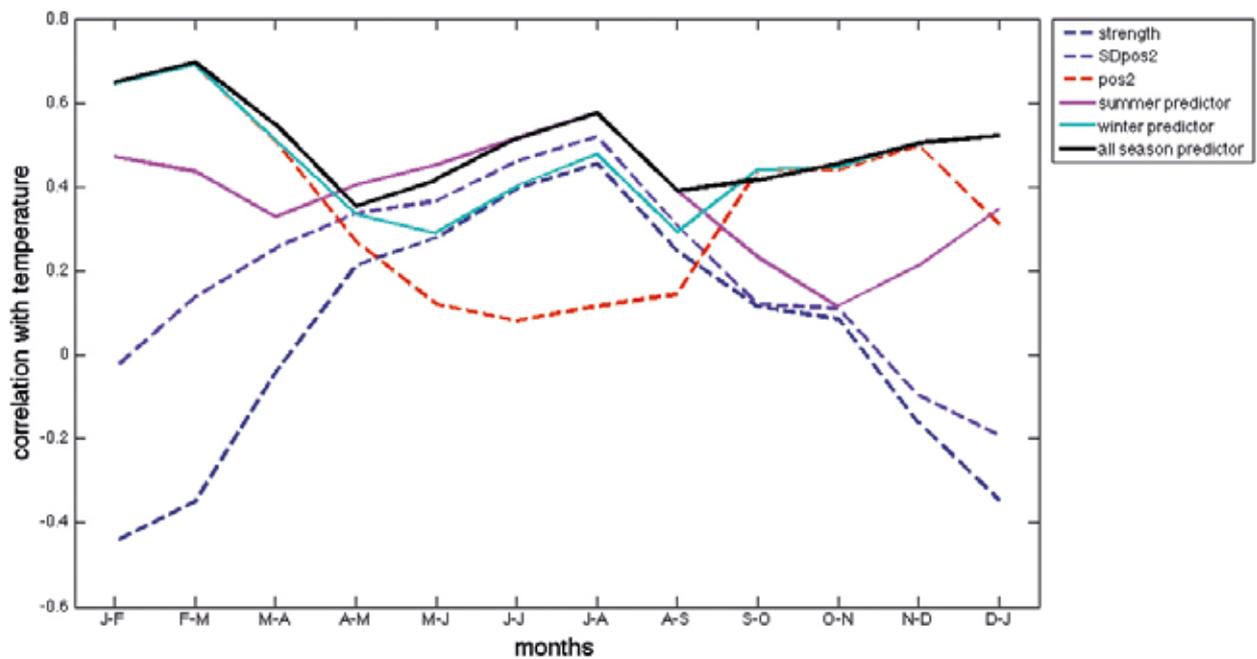
Dr Christopher Nankervis, Founder of Weather Logistics says that the company aim is to tackle food production issues through better management of UK agricultural risks. To achieve this we produce fine scale predictions on 25km space-scales, at challenging 1-6 month 'seasonal' timescales, which include metrics to help predict the growth and health of crops. The vision is to inform grower groups and food procurers on the risks to crop health posed by extreme seasonal weather.

The company's current development is a product to predict the number of growing days. Seasonal forecasts are produced in conjunction with output uncertainties, applying relationships that combine independent datasets from around the globe. This method differs from conventional nested models, with a local forecast produced from its regional parameters. To downscale forecasts to farming scales, Weather Logistics uses a statistical model that combines space-derived data, seasonal forecast output from European weather centres, and historical weather and climate data. The company manages a large amount of post-processed climate data, producing forecasts that describe average weather conditions.



Crown Copyright 2009. The UK Climate Projections data have been made available by the Department for Environment, Food and Rural Affairs (Defra) and Department for Energy and Climate Change (DECC) under licence from the Met Office, Newcastle University, University of East Anglia and Proudman Oceanographic Laboratory. These organisations accept no responsibility for any inaccuracies or omissions in the data, nor for any loss or damage directly or indirectly caused to any person or body by reason of, or arising out of, any use of this data.

The challenge for the ESGI was to discover new avenues to both refine the seasonal forecast and to better quantify its uncertainties. Presenting the problem entitled "How to best combine statistical-empirical relationships to downscale seasonal forecasts?", Weather Logistics attracted a group of enthusiastic industrial mathematicians and were fascinated to discover ways to better select, represent and combine forecast predictors to improve the seasonal forecast output.



Correlation coefficients of individual and combined temperature predictors

Several areas of focus were identified early in the study group; and methods were suggested to tackle each. The study group first set out to understand the forecast concept in more detail, such as the selection of jet stream parameters to form a new diagnostic climate index. Some members of the study group embarked on a mechanistic study to assess the validity of the seasonal forecast process, and devised new techniques to refine and lower uncertainties in numerical predictions. This included applying machine learning techniques to train the forecast with past observations.

The above figure shows the two monthly correlation coefficients of the temperature predictors (smoothing out oscillations seen in one month correlation coefficients). The correlations of the individual predictors (strength, standard deviation and position of the jet stream close to the UK) are shown by the dashed lines; the summer, winter and all season predictors are shown by the solid lines. The summer predictor is a combination

of strength and standard deviation of the jet stream close to the UK, and the winter predictor is a combination of strength and position of the jet stream close to the UK. The all season predictor is formed by taking a weighted sum of summer and winter predictors.

The study group outcome exceeded expectations, providing the company with a comprehensive framework of methods to improve the seasonal forecasts. This will no doubt allow Weather Logistics to make better decisions and perform unbiased calculations of risk for the companies' customers. Furthermore, stimulating the use of mathematics in this context will support the development of new crop strategies and technologies to improve crop health and output efficiency, which are vital for a growing global population compounded by evolving and limited land use.

The 2016 UK Study Group will be held at Durham University from 11th-15th April.

TakeAIM: articulating the influence of mathematics

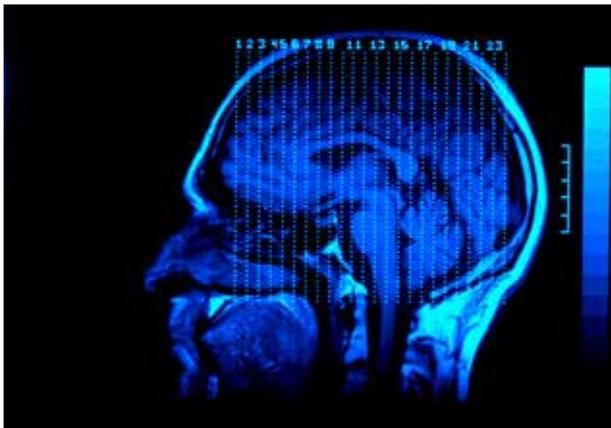


The Smith Institute organises the annual TakeAIM competition, enabling young researchers to make visible the crucial role that mathematics plays and will increasingly play in all aspects of our lives.

Last year's competition attracted 66 entries. The 2015 edition is sponsored by BT, the Satellite Applications Catapult, EPSRC, National Nuclear Laboratory, NAG, Experian, GCHQ, and Babcock International.

Last year's winning entry was submitted by Luca Calatroni, University of Cambridge:

Image Ma-therapy: detect or defect?



A picture of your holidays: nice spot, shining sun, blue sky ... wait! What is that stain occluding your face? No problem: open your favourite photo editor, encircle the stain as best as you can and ... remove it. Easy. Done. Now the scenario changes: the image you are looking at is the MRI of your brain, and there is something strange in it. A doctor is going to analyse it by hand to check if there is something wrong ...

... Do you really feel comfortable thinking that a small defect might be crucial for your diagnosis?

Recent mathematical medical imaging methods provide patients and doctors with accurate, non-invasive tools for the detection of suspected areas in the body. They are helpful also in planning therapies, like radiotherapy: through an accurate segmentation of the region of interest, damages to the surrounding, cancer-free tissues are minimised. For large regions with very sharp boundaries, several, almost fully automatic methods exist, but for low-contrast images with very small regions and fuzzy borders, standard segmentation may fail. We use trained algorithms that, by using examples provided by the user, can learn and compare shapes, intensity and texture information to get the desired region, no matter its properties. Mathematically, this consists in solving diffusion partial differential equations defined on a graph (a pixel image) where the connections between pixels represent their feature similarity. Similar methods can be adapted also in zoology, marine biology and much more.

Trust a mathematician: your brain wants it!

Our achievements

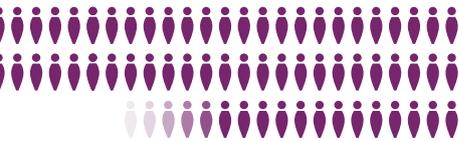
We work across

10
SECTORS



50+

Invited
LECTURES
given



We have elicited

235



student applications for **TakeAIM**



60+

**TECHNICAL
WORKSHOPS**
facilitated

We have supported

13

spectrum auctions across

8

countries



RAISING A TOTAL OF

£9.6bn

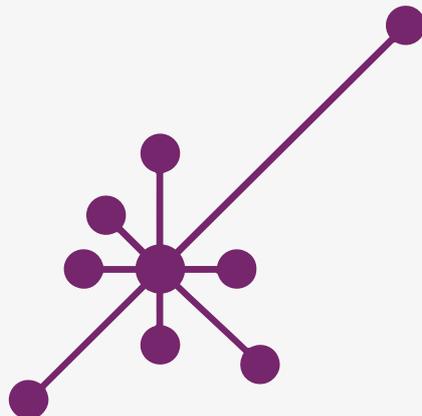


250+

COLLABORATIONS stimulated

60+

MODELS
designed



45+

**CONSULTATIONS
& REVIEWS**



